Intermathematics
Maths Resources for International Schools

Sequences and series mixed questions (SL).

Name……………………..

(1) The first term of an infinite geometric sequence is 6, while the third term is $\frac{2}{3}$. There are two possible sequences. Find the sum of each sequence.

$$6, x, \frac{2}{3}$$

$$\frac{x}{6} = \frac{\frac{2}{3}}{x}$$

$$x^2 = 4$$

$$x = \pm 2$$

$$r = \pm \frac{1}{3}$$

$$S_\infty = \frac{6}{1 - \frac{1}{3}} = 9$$

$$S_\infty = \frac{6}{1 + \frac{1}{3}} = 4.5$$

(2) David goes running every week. He runs 500 metres in the first week. He considers two different training sessions, A and B.

(a) Training session A requires that each week David runs 50 metres more than the previous week and that he continues this for 12 weeks. How far would David run in the final week?

$$u_{12} = 500 + (50)(12 - 1) = 1050$$

(ii) How far would he run over the 12 weeks?

$$S_n = \frac{n}{2}(u_1 + u_n)$$

$$S_n = \frac{12}{2}(500 + 1050) = 9300$$

(b) Training session B requires that each week David runs a distance 10% further than the previous week and that he continues this for 12 weeks. How far would David run in the final week?

$$u_{12} = 500(1.1)^{11} = 1426.558353\ (1430\ 3sf)$$

(ii) How much further in total does David run over the 12 weeks with training session B?

$$S_{12} = \frac{500((1.1)^{12} - 1)}{1.1 - 1} = 10692.14188$$

$$10692.14188 - 9300 = 1390\ (3sf)$$

(3) Find the sum of the infinite geometric series:

$$\frac{1}{5}, \quad -\frac{2}{15}, \quad \frac{4}{45}, \ldots$$

$$S_\infty = \frac{u_1}{1 - r} = \frac{\frac{1}{5}}{1 - -\frac{2}{3}} = 0.12$$

(4) A geometric sequence has all positive terms. The sum of the first two terms is 4 and the sum to infinity is 4.5. Find the value of

(a) The common ratio

$$u_1 + u_1 r = 4$$

$$u_1(1 + r) = 4$$

$$u_1 = \frac{4}{1 + r}$$

$$\frac{u_1}{1 - r} = 4.5$$

$$\frac{\frac{4}{1+r}}{1-r} = 4.5$$

$$4 = 4.5(1-r^2)$$

$$r = \frac{1}{3}$$

(b) The first term.

$$u_1 = \frac{4}{1+\frac{1}{3}} = 3$$

(5) Edgar has $10,000 to invest in a bank offering 3.5% nominal annual interest, compounded monthly.

(a) How much will Edgar have in his account after 5 years?

$$FV = PV\left(1+\frac{r}{100k}\right)^{kn}$$

$$FV = 10{,}000\left(1+\frac{3.5}{100(12)}\right)^{12(5)} = 11909.43$$

(b) How many full years will it take for Edgar to have more than $20,000?

$$FV = 10{,}000\left(1+\frac{3.5}{100(12)}\right)^{12(n)} > 20{,}000$$

$$n > 19.8$$

$$n = 20$$

(6) A scientist measures the population of bacteria in a petri dish over set time intervals. Her results are shown below:

Time, t (hours)	1	2	3
Population, P	50	70	98

(a) If the growth is geometric, find an equation for the population (P) of bacteria after (t) hours.

$$r = \frac{70}{50} = 1.4$$

$$P = 50(1.4)^{t-1}$$

(b) Predict how many bacteria will be in the petri dish after 12 hours.

$$P = 50(1.4)^{12-1} = 2025$$

(c) How long will it take for population of bacteria in the petri dish to be greater than 100,000?

$$P = 50(1.4)^{t-1} > 100{,}000$$

$$t > 23.5$$

$$t = 24$$