Phase portraits and exact solutions of coupled differential equations

(1) You are given the following differential equations.

$$\dot{x} = x - 2y$$

$$\dot{y} = x + dy$$

Show that this system of equations has eigenvalues:

$$\lambda = \frac{1 + d \pm \sqrt{d^2 - 2d - 7}}{2}$$

(b) Hence describe the nature of the solutions to the coupled differential equations near the origin for the following values of d.

(i) $d = -1.5$

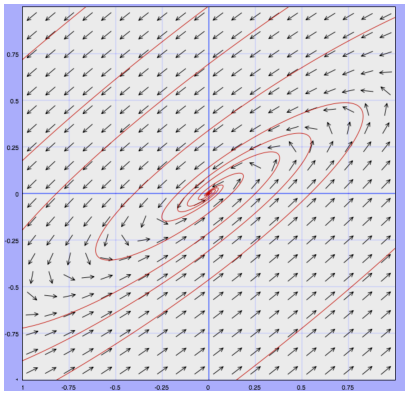
(ii) $d = 0$

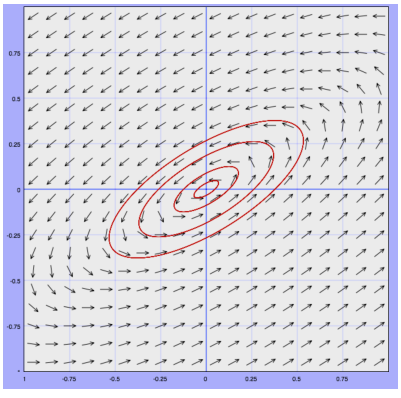
(iii) $d = -1$

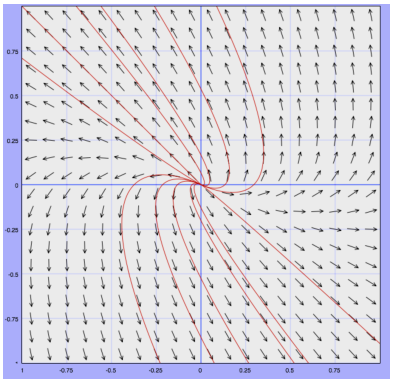
(iv) $d = 4$

(v) $d = -1.9$

(c) Which of the d values found above match the following phase portraits?







(2) You are given the following coupled differential equations:

$$\dot{x} = x + y$$

$$\dot{y} = 3x - y$$

(a) Find the eigenvalues and associated eigenvectors for these equations

(b) Describe the behavior of the solutions near the origin.

(c) Find the exact solution given that $x = 1, y = 2$ when $t = 0$.

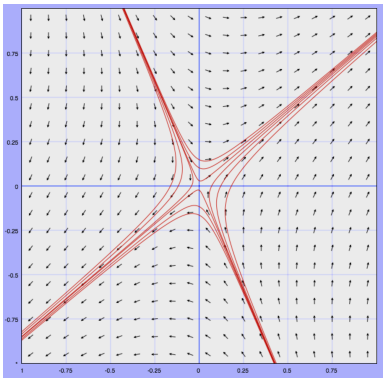
(3) You are given the following coupled differential equations:

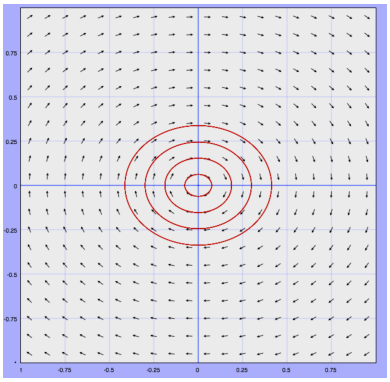
$$\dot{x} = 3y$$

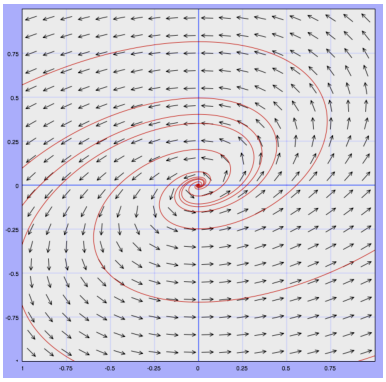
$$\dot{y} = -2x$$

(a) Find the eigenvalues for these equations.

(b) Which of the following phase portraits represents this system?







(4) You are given the following coupled differential equations:

$$\dot{x} = ax + by$$

$$\dot{y} = cx + dy$$

Show that these equations have distinct real eigenvalues when:

$$a^2 + d^2 > 2ad - 4bc$$