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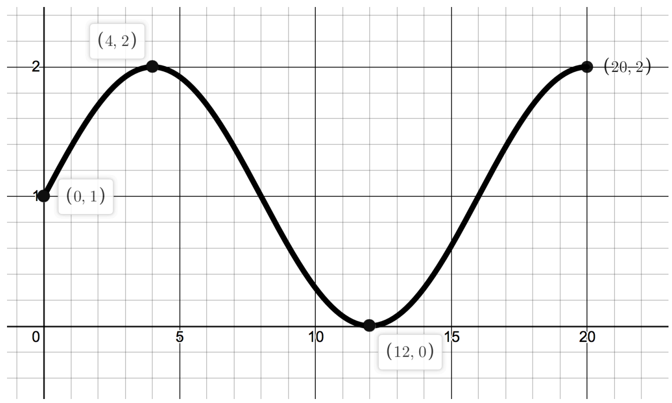
**Understanding graphs: Increasing/decreasing functions and stationary points**

**Name……………………..**

(1) A graph of a function $f(x)$ is shown below:




(a) For the graph $f(x)$, when is $f'(x) < 0$?

$$4 < x < 12$$

(b) When is $f'(x) = 0$?

$$x = 4, 12, 20$$

(2) A function $f(x)$ is defined below:

$$f(x) = \frac{5}{x^3}, x \neq 0,$$

(a) Write down the equation of the vertical and horizontal asymptotes.

$$x = 0, \qquad y = 0$$

(b) Show using calculus that $f(x)$ is a decreasing function for all $x$ in its domain.

$$f'(x) = -15x^{-4}$$

$$x^{-4} > 0 \text{ in domain}$$

$$\text{Therefore } f'(x) < 0$$

(3) A function $f(x)$ is defined below:

$$f(x) = 5ax^2 + x$$

For what values of $x$ is $f(x)$ a decreasing function?

$$f'(x) = 10ax + 1 < 0$$

$$x < -\frac{1}{10a}$$

(4) Use the **first derivative test** to classify any stationary points of the following graphs:

(a) $f(x) = 3x^2 + 6x - 2$

(a) $f'(x) = 6x + 6 = 0$

$x = -1$

Min at -1

(b) $f(x) = -x^3 + 6x^2 + 36x - 2$

(b) $f'(x) = -3x^2 + 12x + 36 = 0$

$x = 6, -2$

min at -2, max at 6.

(c) $f(x) = \frac{x^3}{3} - x^2 - 3x + 4$

(c) $f'(x) = x^2 - 2x - 3 = 0$

$x = -1, 3$

Max at -1, min at 3

(d) $f(x) = \frac{x^3}{3} - x^2 - 8x - 1$

(d) $f'(x) = x^2 - 2x - 8 = 0$

$x = 4, -2$

max at -2, min at 4.

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(5) Use the **second derivative test** to classify any stationary points of the following graphs:

(a) $f(x) = 3x^3 + 6x^2$

$f'(x) = 9x^2 + 12x = 0$

$x = 0, -\frac{4}{3}$

$f''(x) = 18x + 12$

$f''(0) > 0$ Min

$f''\left(-\frac{4}{3}\right) < 0$ Max

(b) $f(x) = \frac{x^4}{2} - x^2 + 6$

$f'(x) = 2x^3 - 2x = 0$

$x = 0, 1, -1$

$f''(x) = 6x^2 - 2$

$f''(0) < 0$ Max

$f''(1) > 0$ Min

$f''(-1) > 0$ Min

(c) $f(x) = 3x^3 - x$

$f'(x) = 9x^2 - 1 = 0$

$x = \pm\frac{1}{3}$

$f''(x) = 18x$

$f''\left(\frac{1}{3}\right) > 0$ Min

$f''\left(-\frac{1}{3}\right) < 0$ Max

(d) $f(x) = x^3 - 2x^2 - 4x + 9$

$f'(x) = 3x^2 - 4x - 4 = 0$

$x = 2, -\frac{2}{3}$

$f''(x) = 6x - 4$

$f''(2) > 0$ Min

$f''\left(-\frac{2}{3}\right) < 0$ Max

(6) Classify any stationary points of the following graph:

$$y = \frac{x^2}{1+x}$$

$$\frac{dy}{dx} = \frac{x^2+2x}{(1+x)^2} = 0$$

$$x^2 + 2x = 0$$

$$x = 0\ (Min)\ , x = -2\ (\text{Max})$$

(first differential test easier here)

(7) The graph $f(x) = x^3 + ax^2 - bx + 10$ has a stationary point at $(1,2)$.

(a) Find $a, b$.

$$f(1) = (1)^3 + a(1)^2 - b(1) + 10 = 2$$

$$f'(1) = 3(1)^2 + 2a(1) - b = 0$$

$$a - b = -9$$

$$2a - b = -3$$

$$a = 6, b = 15$$

(b) Classify all stationary points.

$$f'(x) = 3(x)^2 + 12(x) - 15 = 0$$

$$x = 1, -5$$

$$f''(x) = 6x + 12$$

$$x = 1\ \ (\text{min})$$
$$x = -5\ \ (\text{max})$$

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