Functions 10: Mixed exam style questions [non GDC]

(1) The following functions are defined:

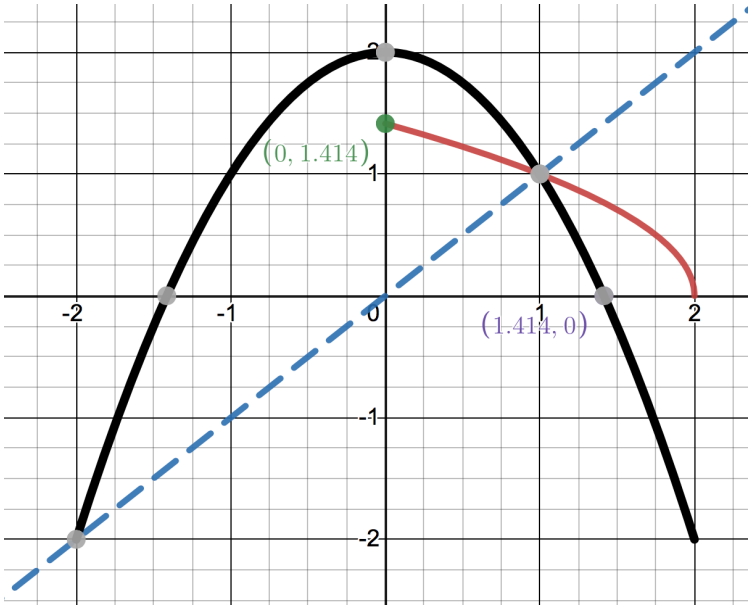
$$f(x) = x^2 \quad g(x) = -x + 2 \quad h(x) = x + 1$$

(a) Sketch $gf(x)$ below $(-2 \leq x \leq 2)$. Describe the transformation that takes $f(x)$ to $gf(x)$.

$$gf(x) = -x^2 + 2$$

Reflection in x axis, translation $\begin{pmatrix} 0 \\ 2 \end{pmatrix}$

(b) $f(x)$ is now restricted to the domain $0 \leq x \leq 2$. Sketch $f^{-1}(x)$.



(2) The function $f(x) = \frac{ax+3}{2x-b}$ has asymptotes with equations $x = 3$ and $y = 4$.

(a) Find a and b.

$$2(3) - b = 0$$
$$b = 6$$

$$\frac{a}{2} = 4$$

$$a = 8$$

(b) Find the inverse function $f^{-1}(x)$.

$$f(x) = \frac{8x+3}{2x-6}$$

$$x = \frac{8y+3}{2y-6}$$

$$2xy - 6x = 8y + 3$$

$$y(2x-8) = 3 + 6x$$

$$f^{-1}(x) = \frac{3+6x}{2x-8}$$

(c) The function $g(x)$ is defined as: $g(x) = \frac{2(ax+3)}{2x-b} + 2$. Describe the set of transformations which take $f(x)$ to $g(x)$.

Vertical stretch factor 2, translation $\begin{pmatrix} 0 \\ 2 \end{pmatrix}$

(3) For what values of m does $f(x) = 2x^2 + mx + m$ have no real roots?

$$b^2 - 4ac < 0$$

$$m^2 - 4(2)(m) < 0$$

$$m^2 - 8m < 0$$

$$m(m-8) < 0$$

$$0 < m < 8$$

(4) For $f(x) = x^2 + 2x + 1$ and $g(x) = 3x + c$, for what values of c do the graphs intersect in 2 points?

$$b^2 - 4ac > 0$$

$$x^2 + 2x + 1 = 3x + c$$

$$x^2 - x + 1 - c = 0$$

$$1 - 4(1 - c) > 0$$

$$-3 + 4c > 0$$

$$c > \frac{3}{4}$$

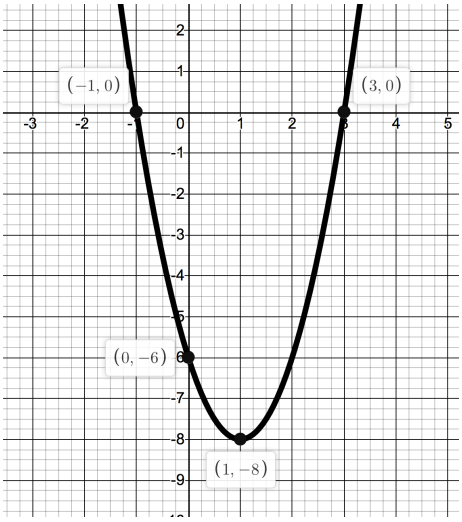
(5) The function $f(x)$ is defined as: $f(x) = 2x^2 - 4x - 6$.

(a) Rewrite the quadratic **both** in the form $y = r(x - p)^2 + q$ and $y = (ax + b)(cx + d)$.

$$f(x) = 2(x - 1)^2 - 8$$

$$f(x) = (2x + 2)(x - 3) \ \ or \ (x + 1)(2x - 6)$$

(b) Hence sketch the graph showing axes intercepts and coordinates of the vertex.



(c) Find the coordinates of the vertex for the function $f(x + 1) + 2$

Translation $\begin{pmatrix} -1 \\ 2 \end{pmatrix}$ therefore $(0, -6)$

(6) Some functions are defined below:

$$f(x) = 2x - 2, \qquad g(x) = x^2 - x \qquad h(x) = \frac{x+2}{x-1}, x \neq 1$$

(a) Find $gf(x) = 12$

$$(2x-2)^2 - (2x-2) = 12$$

$$4x^2 - 8x + 4 - 2x + 2 = 12$$

$$4x^2 - 10x - 6 = 0$$

$$x = 3, \qquad x = -0.5$$

(b) Find $h^{-1}(3)$

$$x = \frac{y+2}{y-1}$$

$$xy - x = y + 2$$

$$y(x-1) = 2 + x$$

$$h^{-1}(x) = \frac{2+x}{x-1}$$

$$h^{-1}(3) = 2.5$$

(c) Find $ff(x) = f^{-1}(x)$

$$ff(x) = 2(2x-2) - 2 = 4x - 6$$

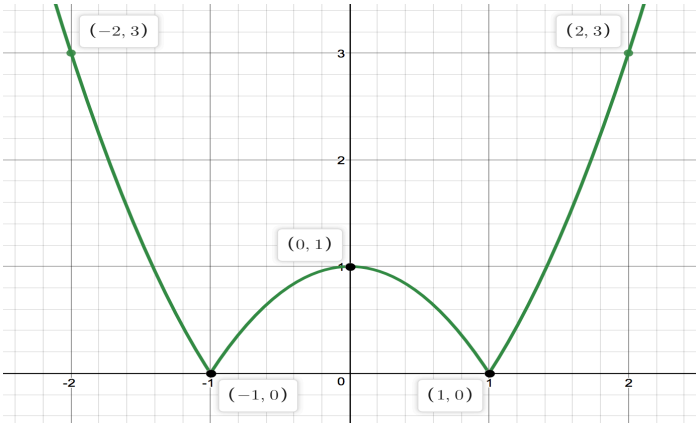
$$f^{-1}(x) = \frac{x+2}{2}$$

$$\frac{x+2}{2} = 4x - 6$$

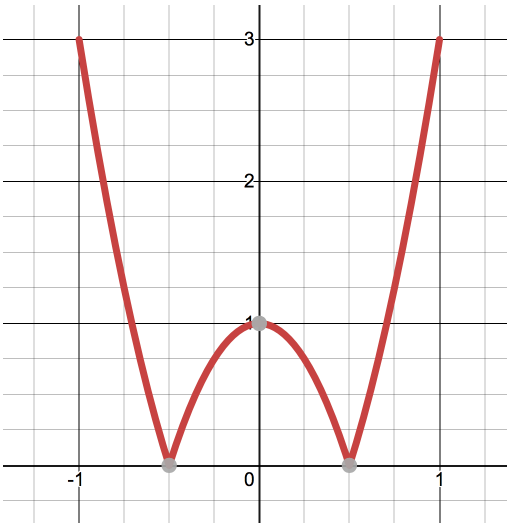
$$x + 2 = 8x - 12$$

$$x = 2$$

(7) Below is the graph of $f(x)$.



(a) Sketch the graph of $f(2x)$. $(-1 \leq x \leq 1)$.



(b) Sketch the graph of $f(x-1)+1$. $(-1 \leq x \leq 3)$.