Intermathematics
Maths Resources for International Schools

Starter of the Week 6: The sum of three cubes

(a) How many of these can you form as the sum of 3 cubes? You can use negative integers if you wish.

1	
2	$1^3 + 0^3 + 1^3$
3	
6	
7	$(-1)^3 + 2^3 + 0^3$
8	
9	
10	
11	
12 [hard!]	
15	
16	
17	
18	
19	
21 [hard!]	

(b) Research what the solution is for 30 and 33!

(c) Can you find a pattern that describes the numbers that are missing from this list?

Teacher notes:

Level of difficulty: Accessible for all

Syllabus knowledge required: None

This would be a good starter which could either be done when studying proof in Analysis.

The sum of 3 cubes problem is an unsolved problem in mathematics. It is hypothesized that every positive integer, k not of the form $9q \pm 4, q \in \mathbb{Z}$ can be written as:

$$a^3 + b^3 + c^3 = k \qquad a, b, c, \in \mathbb{Z}$$

(1) How many of these can you form as the sum of 3 cubes? You can use negative integers if you wish.

1	$0^3 + 0^3 + 1^3$
2	$1^3 + 0^3 + 1^3$
3	$1^3 + 1^3 + 1^3$
6	$(-1)^3 + 2^3 + (-1)^3$
7	$(-1)^3 + 2^3 + 0^3$
8	$2^3 + 0^3 + 0^3$
9	$2^3 + 1^3 + 0^3$
10	$2^3 + 1^3 + 1^3$
11	$(-2)^3 + 3^3 + (-2)^3$
12 [hard!]	$(-11)^3 + 10^3 + 7^3$
15	$2^3 + 2^3 + (-1)^3$
16	$2^3 + 2^3 + 0^3$
17	$2^3 + 2^3 + 1^3$
18	$(-2)^3 + 3^3 + (-1)^3$
19	$(-2)^3 + 3^3 + 0^3$
21 [hard!]	$(-11)^3 + 16^3 + (-14)^3$

(b) The solutions for 30 and 33 are shown below. The one for 33 was only solved in 2019.

$$30 = (2,220,422,932)^3 + (-2,218,888,517)^3 + (-283,059,965)^3$$

$$33 = 8\,866\,128\,975\,287\,528^3 + (-8\,778\,405\,442\,862\,239)^3 + (-2\,736\,111\,468\,807\,040)^3.$$

(c) The numbers not on the list are of the form $9q \pm 4, q \in \mathbb{Z}$.