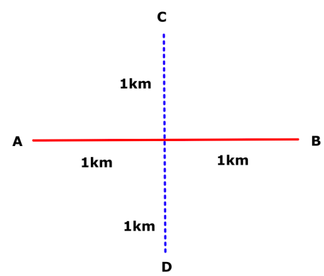
Avoiding a magical barrier. [30 marks]

1. *[Maximum marks: 30]*

In this question we explore a scenario where we walk from Town A to Town B, a distance of 2km.
50% of the time there is no obstruction along this route. However, 50% of the time there is a magical barrier perpendicular to the route exactly half way between A and B, extending for 1km in both directions. This barrier is invisible and can only be sensed when you meet it.



Your task is to investigate the optimum strategy for minimizing your average journey.

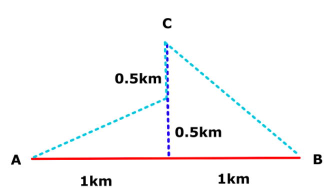
(a) *You set out directly on the route AC. You then walk in the line CB. Find the distance travelled.*

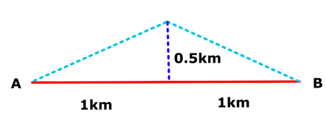
[1]

(b) *You set out directly on the route AB. If there is no barrier you continue on this route to B. If you meet the barrier you walk up to C and then in the line CB. Find the average distance travelled.*

[3]

(c) *This time you set off from A in a straight line to a point 0.5 km along the perpendicular bisector of AB. If you meet the barrier you continue up to C before travelling in the line CB. If you don't meet the barrier you head straight for point B.*

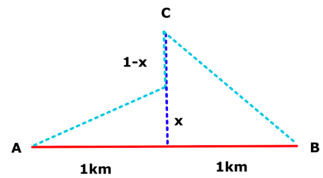


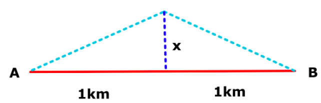


Find the average distance travelled.

[3]

(d) *This time you set off from A in a straight line to a point x km along the perpendicular bisector of AB. If you meet the barrier you continue up to C before travelling in the line CB. If you don't meet the barrier you head straight for point B.*





(i) Find an equation for the average distance travelled in terms of x.

[3]

(ii) Use ***calculus*** *to find the exact value of x which minimizes the average distance travelled.*

[5]

(iii) Sketch a graph to verify your result graphically. What is the minimum average distance?

[2]

(e) *The barrier now appears n % of the time. Use calculus to find the optimum distance x, in terms of n.*

[7]

(f) *For a given integer value of n, the optimum strategy is simply to head in a straight line from A to C. Find this value of n.*

[6]