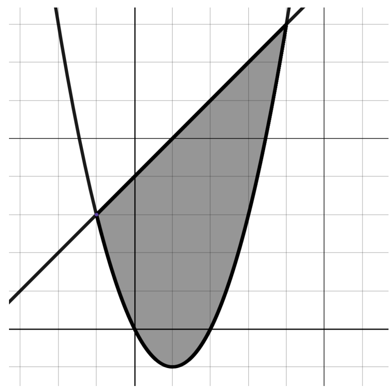
Areas between curves and Fundamental Theorem of Calculus

Name……………………….

(1) The graph below shows the line $y = x + 4$ and the curve $y = x^2 - 2x$



(a) Find the x-coordinates of the intersection between the line and the curve.

$$x^2 - 2x = x + 4$$

$$x^2 - 3x - 4 = 0$$

$$x = -1, \qquad x = 4$$

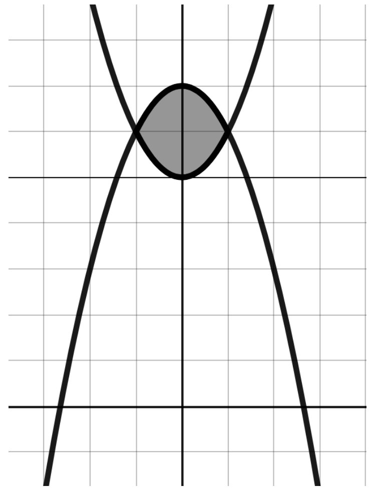
(b) Hence find the shaded area.

$$\int_{-1}^{4} x + 4 - (x^2 - 2x)dx$$

$$\int_{-1}^{4} -x^2 + 3x + 4dx = \left[-\frac{x^3}{3} + \frac{3x^2}{2} + 4x\right]_{-1}^{4}$$

$$= \frac{125}{6}$$

(2) The graph below shows the line $y = -x^2 + 7$ and the curve $y = x^2 + 5$



Find the shaded area.

$$-x^2 + 7 = x^2 + 5$$

$$2x^2 + 2$$

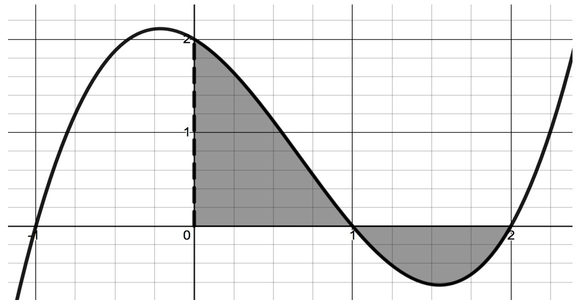
$$x = \pm 1$$

$$\int_{-1}^{1} -x^2 + 7 - (x^2 + 5)dx$$

$$\int_{-1}^{1} -2x^2 + 2dx = \left[-\frac{2x^3}{3} + 2x\right]_{-1}^{1}$$

$$= \frac{8}{3}$$

(3) A cubic graph is shown below:



(a) Show that the equation of this graph is $y = x^3 - 2x^2 - x + 2$

$$y = a(x+1)(x-1)(x-2)$$

$$y = a(x^2-1)(x-2)$$

$$y = a(x^3 - 2x^2 - x + 2)$$

When $x = 0, y = 2$. Therefore $a = 1$

$$y = x^3 - 2x^2 - x + 2$$

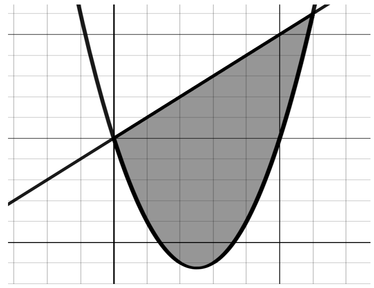
(b) Find the total shaded area.

$$\int_0^1 x^3 - 2x^2 - x + 2dx = \frac{13}{12}$$

$$\int_1^2 x^3 - 2x^2 - x + 2dx = -\frac{5}{12}$$

$$\frac{13}{12} + \left|-\frac{5}{12}\right| = \frac{3}{2}$$

(4) The graph below shows the line $y = x + 5$ and the curve $y = x^2 - 5x + 5$



Find the total shaded area

$$x^2 - 5x + 5 = x + 5$$

$$x^2 - 6x = 0$$

$$x = 0, \qquad x = 6$$

$$\int_0^6 x + 5 - (x^2 - 5x + 5)dx$$

$$\int_0^6 -x^2 + 6x\,dx = \left[-\frac{x^3}{3} + 3x^2\right]_0^6 = 36$$

(5) Given that:

$$\int_{-1}^{3} f(x)dx = 8, \quad \int_{-1}^{5} f(x)dx = 2, \quad \int_{-1}^{5} g(x)dx = 5,$$

Find:

(a) $\int_{-1}^{5} f(x) + 2g(x)\, dx$

$2 + 2(5) = 12$

(b) $\int_{3}^{5} f(x)dx$

$\int_{-1}^{5} f(x)dx - \int_{-1}^{3} f(x)dx$

$2 - 8 = -6$

(c) $\int_{-1}^{5} f(x) + 2\, dx,$

$$2 + \int_{-1}^{5} 2\, dx = 14$$

(d) $\int_{1}^{5} f(x-2)\, dx$

$$\int_{1}^{5} f(x-2)\, dx \quad = \int_{-1}^{3} f(x)\, dx \quad = 8$$