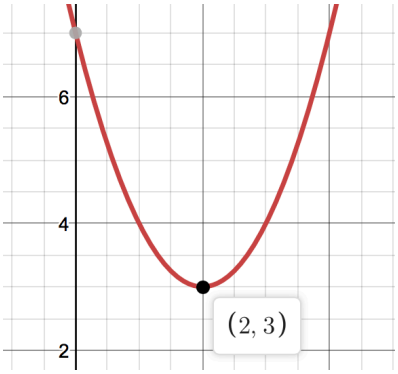
Intermathematics
Maths Resources for International Schools

Functions 1: Functions and their inverse

Name……

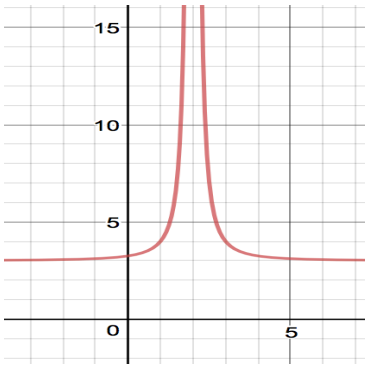
1) Find the domain and range of the following functions:

(a)



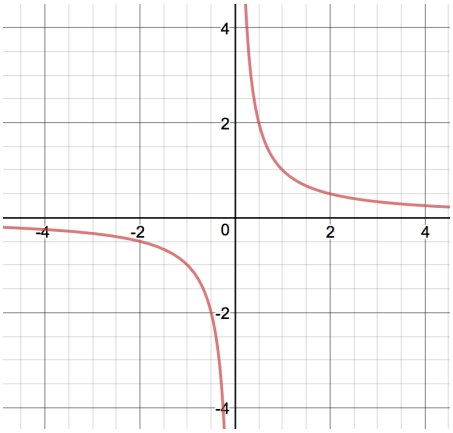
(a) $x \in \mathbb{R}, y \geq 3$

(b)



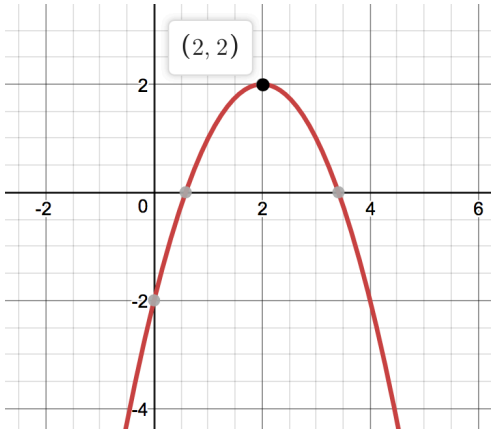
(b) $x \in \mathbb{R}\ x \neq 2,\ y > 3,$

(c)



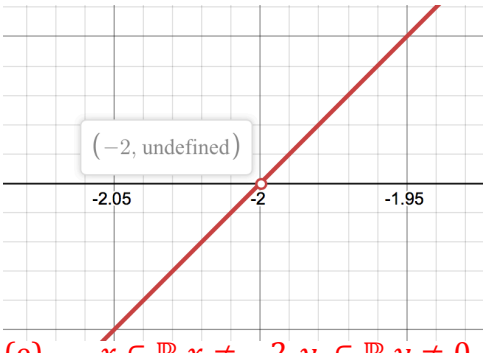
(c) $x \in \mathbb{R}\ x \neq 0, y\ \in \mathbb{R}\ y \neq 0$

(d)



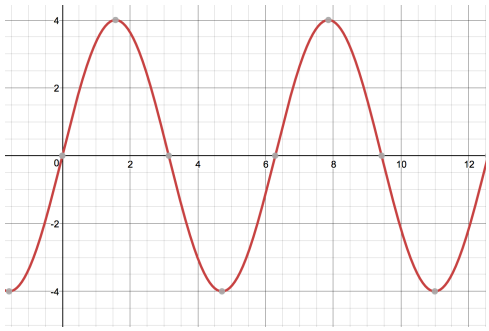
(d) $x \in \mathbb{R}\ y \leq 2,$

(e)



(e) $x \in \mathbb{R}\ x \neq -2, y\ \in \mathbb{R}\ y \neq 0$

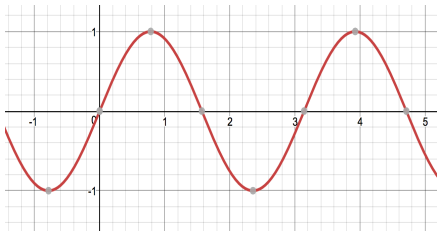
(f)



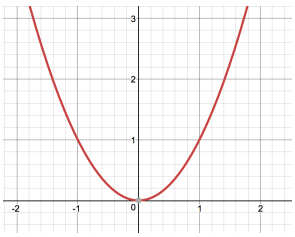
(d) $x \in \mathbb{R} - 4 \leq y \leq 4,$

2) Do the following functions have an inverse?

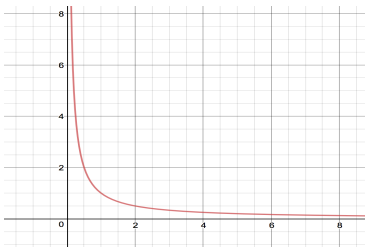
(a)



(b)



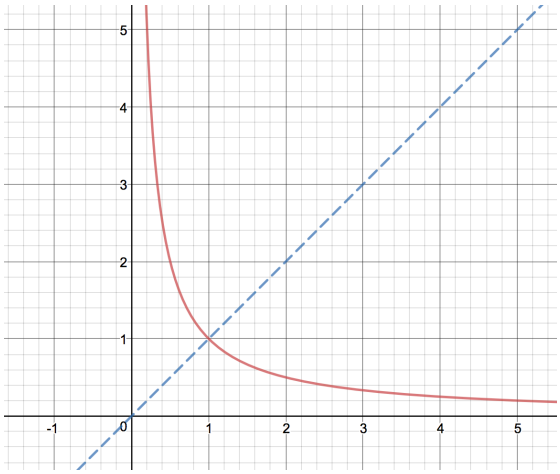
(c)



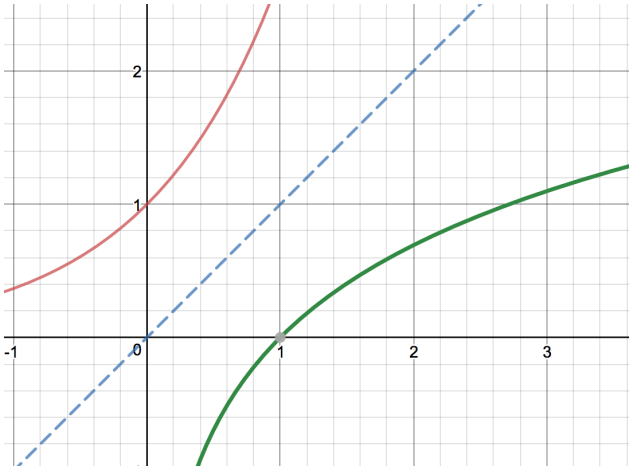
(a) No – fails horizontal test (b) No – fails horizontal test (c) Yes.

3) Sketch the inverse of the following graphs using the line y = x to help:
The inverse graphs are shown in green. (a) is a self inverse and so is the same graph.

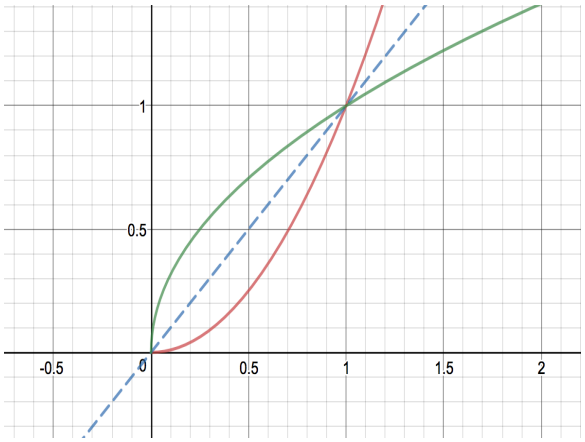
(a)



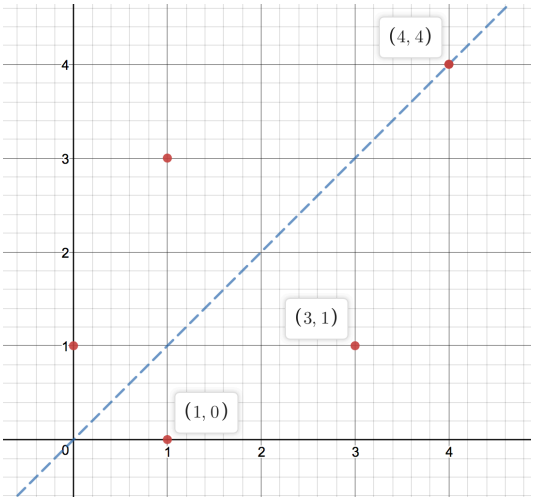
(b)



(c)



(d)



(4) The following functions are defined as:

$$f(x) = 4x + 2, \qquad g(x) = x^2 + x \qquad h(x) = \frac{x}{x+2}, x \neq -2$$

Find:

(a) $f(2)$ (b) $fg(x)$ (c) $gf(3)$ (d) $f^{-1}(x)$

(a) 10 (b) $4x^2 + 4x + 2$ (c) 210 (d) $f^{-1}(x) = \frac{x-2}{4}$

(e) $h^{-1}(x)$ (f) $hh^{-1}(x)$ (g) $f(x+1)$

(e) $x = \frac{y}{y+2}$ (f) x (g) $4(x+1) + 2 = 4x + 6$

$yx + 2x = y$

$y(x-1) = -2x$

$h^{-1}(x) = \frac{-2x}{x-1}$

(5) The following functions are defined as:

$$f(x) = -2x + 7, \qquad g(x) = x^2 + 8x + 23$$

(a) If $f(x) = 10$. Find x.

$$-2x + 7 = 10$$
$$x = -1.5$$

(b) If $f(x) = g(x)$. Find x.

$$-2x + 7 = x^2 + 8x + 23$$

$$0 = x^2 + 10x + 16$$

$$x = -8, x = -2$$

(6) You are given the following information:

$$f(x) = \frac{a}{bx+3} \qquad f(3) = \frac{1}{33} \qquad f^{-1}(2) = -\frac{1}{4}$$

(a) Find a and b.

$$x = \frac{a}{by+3}$$

$$x(by+3) = a$$

$$xby + 3x = a$$

$$f^{-1}(x) = \frac{a-3x}{bx}$$

$$f^{-1}(2) = \frac{a-3(2)}{b(2)} = -\frac{1}{4}$$

$$a = -0.5b + 6$$

$$f(3) = \frac{a}{b(3)+3} = \frac{1}{33}$$

$$a = \frac{1}{11}b + \frac{1}{11}$$

$$\frac{16}{15}b = -\frac{2}{3}$$

$$b = 10, a = 1$$

(b) What is the domain of $f(x)$?

$$f(x) = \frac{1}{10x+3}$$

$$x \in \mathbb{R}\, x \neq -0.3$$

(7) The following function is defined as:

$$f(x) = \frac{2x+3}{3-2x}, x \neq \frac{3}{2}$$

(a) Find the equation of the horizontal and vertical asymptotes.

$$x = 1.5$$
$$y = -1$$

(b) Find the coordinates of where $f(x)$ crosses the x-axis and the y-axis.

$$f(0) = \frac{2(0)+3}{3-2(0)} = 1$$

$$(0,1)$$

$$f(x) = \frac{2x+3}{3-2x} = 0$$

$$x = -\frac{3}{2}$$

$$(-\frac{3}{2}, 0)$$

(c) Draw a sketch of the graph of $f(x)$.