Sequence and Series 6: Mixed questions (HL)

(1) A 125.25 metre rope is cut into n pieces of increasing lengths that form an arithmetic sequence with a common difference of d metres. Given that the lengths of the shortest and longest pieces are 2 metres and 6.35 metres respectively, find the values of n and d.

(2) Find the sum of all three-digit natural numbers that are not exactly divisible by 5.

(3) The mean of the first ten terms of an arithmetic sequence is -9.5. The mean of the first twenty terms of the arithmetic sequence is -24.5. Find the value of the 10^{th} term of the sequence.

(4) An arithmetic sequence has first term a and has common difference d, $d \neq 0$. The 2^{nd}, 5^{th}, 14^{th} terms of the arithmetic sequence are the first three terms of a geometric sequence.

(a) Show that $a = 0.5d$.

(b) Given that the sum of the first 10 terms of the arithmetic sequence is 200, find the sum of the first 10 terms of the geometric sequence.

(5) The common ratio of the terms in a geometric series is 3^x

(a) State the domain of values of x for which the sum to infinity of the series exists.

(b) If the first term of the series is 60, find the value of x for which the sum to infinity is 80.

(6) A geometric sequence has a first term of 5 and a common ratio of 1.2. Find the number of terms needed such that the sum of the series is greater than 1000.

(7) On the first of January 2000, Fred decides to invest \$500. He invests in a bank offering 2% annual compound interest. On the first of January each following year he invests a further \$100.

(a) Show that on the first of January 2002 he will have \$722.20 in his account.

(b) How much money will be in his account on the first of January 2020?

(8) A box contains 3 white balls and 1 black ball. Alice and Bob take it in turns to choose a ball. The ball is then replaced. The first person to choose a black ball wins. Alice goes first. What is the probability that Alice wins?