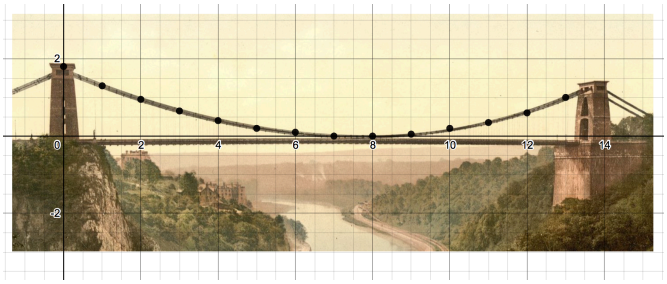
Intermathematics
Maths Resources for International Schools

Building bridges

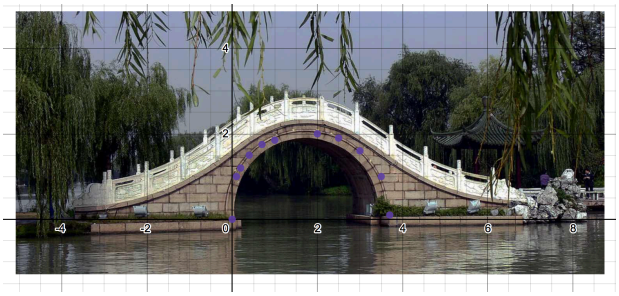
In this investigation we will look at the shape of 3 different bridges and decide how well a parabola fits their curves – and whether another function provides a better fit. The 3 bridges are:

A. The Clifton Suspension bridge



https://commons.wikimedia.org/wiki/File:Clifton_Suspension_Bridge_c1900_2.jpg

B. Twenty four Bridge in Yangzhou



https://en.wikipedia.org/wiki/Moon_bridge#/media/File:Twenty_four_bridge.JPG

C. The Mathematical Bridge at Queens Cambridge

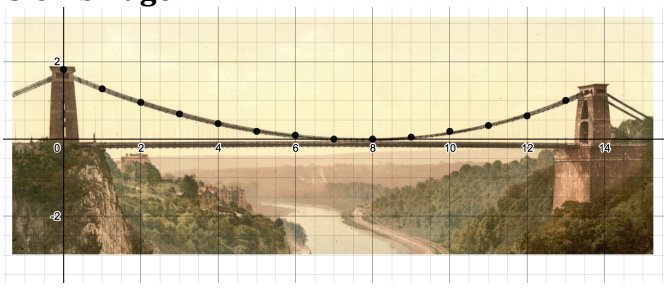


https://www.queens.cam.ac.uk/files/mathematical_bridge_0.jpg

1. Look at the 3 pictures above and decide which of these can be described by a parabola. Can you think of any other equations which might fit these curves?

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A. The Clifton Suspension bridge



I imported the above picture into Desmos and chose some points that described the general curve. Note that the x and y axes are not scaled as we are simply interested in the shape. This gives the following data.

x	y
0	1.8
1	1.3
2	0.95
3	0.65
4	0.4
5	0.2
6	0.1
7	0
8	0
9	0.05
10	0.2
11	0.35
12	0.6
13	1

A1. Plot these points on Desmos and fit a curve of the form:

$$y = a(x - p)^2 + q$$

One possible coordinate to choose as the minimum is (7.5,0)

$$y = a(x - 7.5)^2 + 0$$

Choosing another coordinate, (0, 1.8)

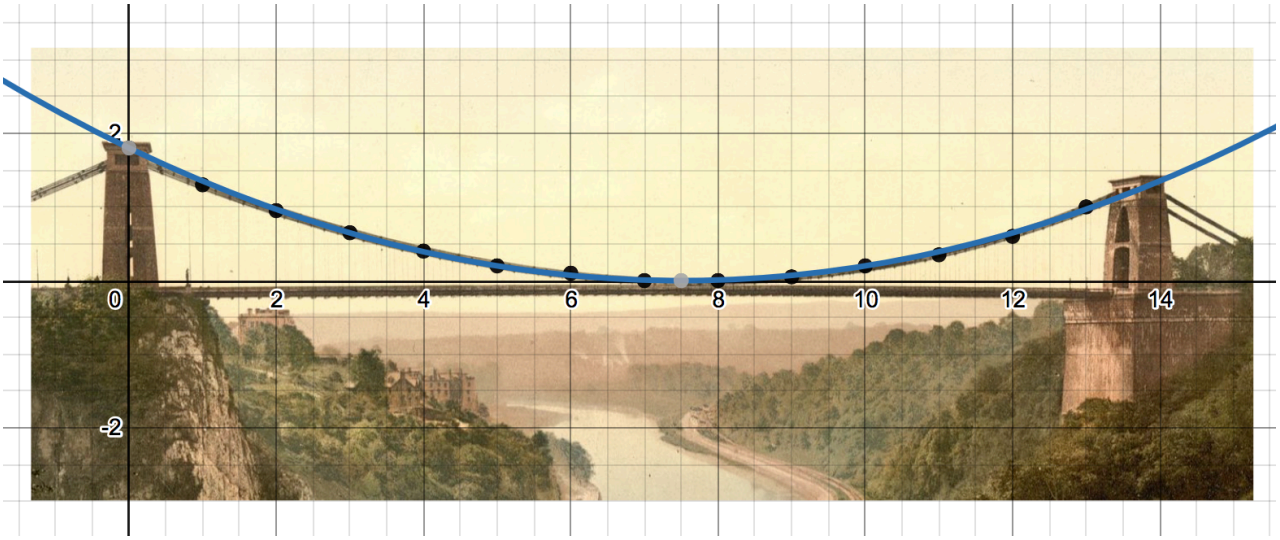
$$1.8 = a(0 - 7.5)^2 + 0$$

$$a = \frac{1.8}{56.25}$$

$$y = 0.032(x - 7.5)^2$$

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This gives:



A2. Use the Desmos regression tool to find a quadratic regression line. Compare this regression line to your answer for (2).

$$y_1 \sim a(x_1 - b)^2 + c$$

STATISTICS

$R^2 = 0.9979$

RESIDUALS

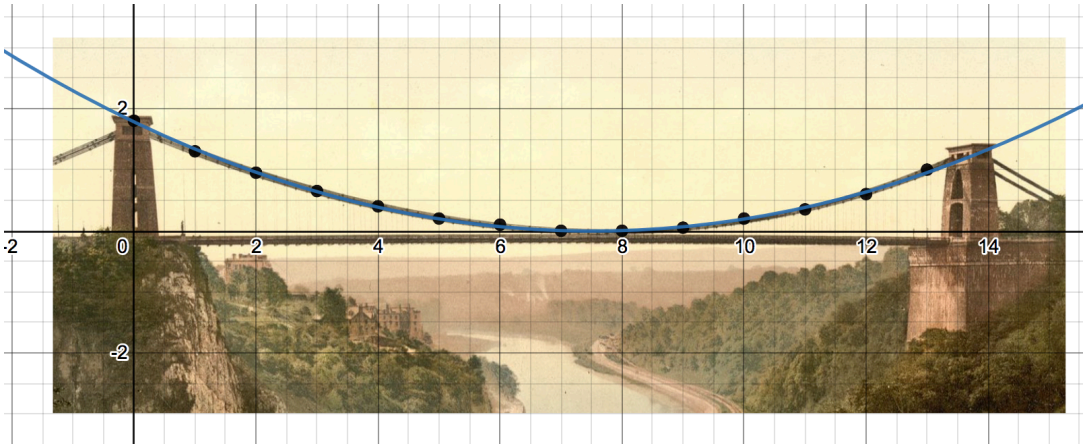
e_1 plot

PARAMETERS

$a = 0.0316964$ $b = 7.50888$

$c = -0.00447194$

This gives a very similar graph:

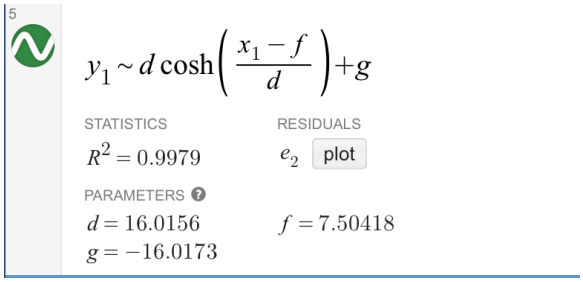


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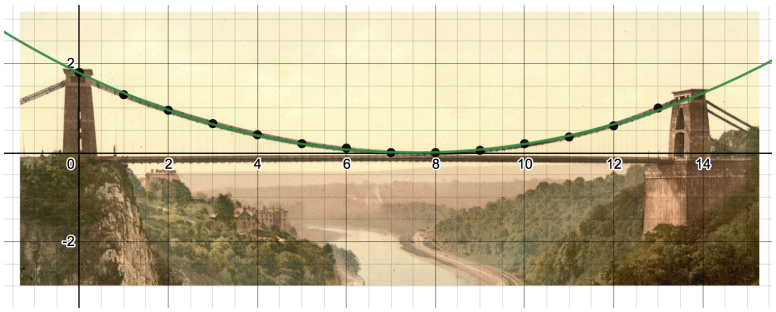
A3. The ropes on a suspension bridge should hang in the form of a Catenary curve.

$$y = \text{acosh}\left(\frac{x-b}{a}\right) + c$$

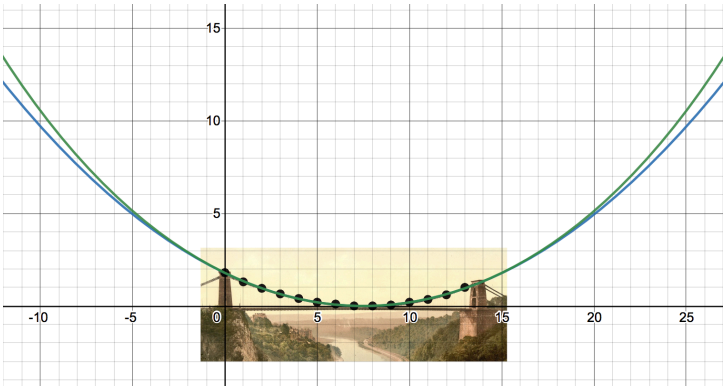
(Here $cosh(x)$ is the hyperbolic cosine which you might like to research).
Use Desmos to find a regression line with this form. Compare your result to (3).



This gives:

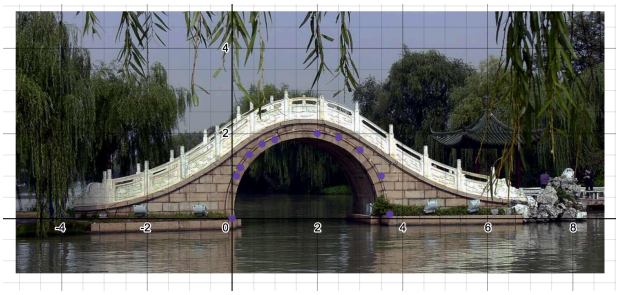


Both the quadratic and the Catenary fit very well (and both have an identical R^2value). Plotting these 2 curves together we can see that on the scale of our image they are almost indistinguishable, but over a larger scale we can start to see the difference in shape.



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B. Twenty four Bridge in Yangzhou



Following the same method as for A we get the following data:

x	y
0	0
0.1	1
0.2	1.2
0.4	1.5
0.7	1.75
1	1.85
2	2
2.5	1.9
3	1.6
3.5	1
3.7	0.1

B1. Plot these points on Desmos and fit a curve of the form:

$$y = a(x - p)^2 + q$$

One possible coordinate to choose as the minimum is $(1.7, 2.05)$

$$y = a(x - 1.7)^2 + 2.05$$

Choosing another coordinate, $(0, 0)$

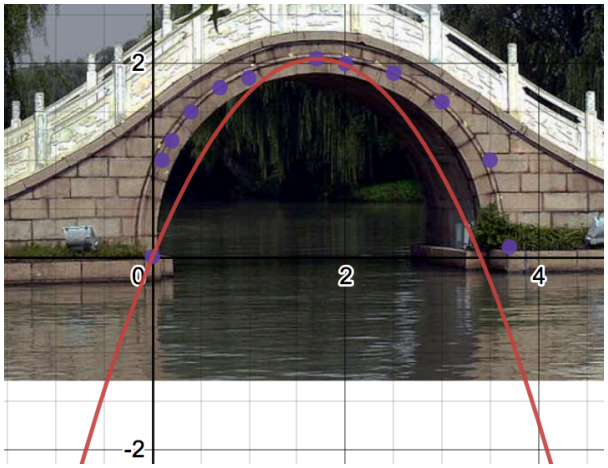
$$0 = a(0 - 1.7)^2 + 2.05$$

$$a = \frac{-2.05}{2.89}$$

$$y = -0.709(x - 1.7)^2 + 2.05$$

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This gives:



B2. Use the Desmos regression tool to find a quadratic regression line. Compare this regression line to your answer for (2).

$$y_1 \sim a(x_1 - p)^2 - q$$

STATISTICS

$R^2 = 0.8418$

RESIDUALS

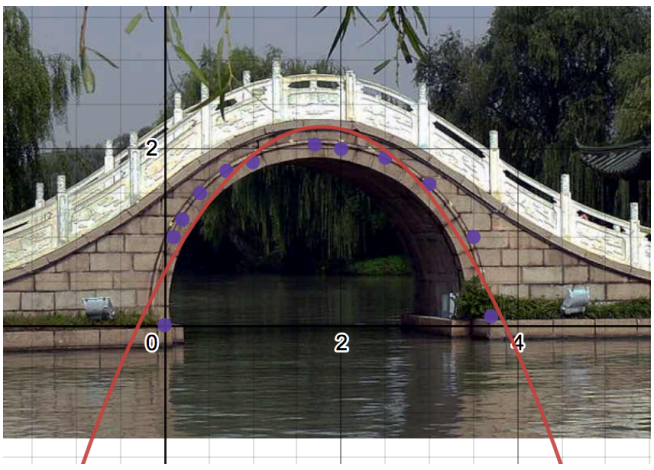
e_1 plot

PARAMETERS

$a = -0.521328$ $p = 1.78124$

$q = -2.25286$

This gives:



This is better than our attempt – but clearly does not fully capture this curve.

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B3. The moon bridge should follow a semi-circular equation of the form:

$$y = \sqrt{r^2 - (x - a)^2} + b$$

Use Desmos to find a regression line with this form. Compare your result to (3).

$$y_1 \sim \left(r^2 - \left(x_1 - c\right)^2\right)^{0.5} + d$$

STATISTICS
$R^2 = 0.9057$

RESIDUALS
e_2 plot

PARAMETERS
$r = -1.88539$
$c = 1.81461$
$d = 0.110856$

This gives:



We can see that a semi circle fits these data points much better.

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C. The Mathematical Bridge at Queens Cambridge



Following the same method as for A and B we get the following data:

x	y
0	0
1	0.68
2	1.2
3	1.6
4	1.9
5	2.1
6	2.2
7	2.2
8	2.15
9	1.95
10	1.7
11	1.3
12	0.85
13	0.2

C1. Plot these points on Desmos and fit a curve of the form:

$$y = a(x - p)^2 + q$$

One possible coordinate to choose as the minimum is $(6.5, 2.25)$

$$y = a(x - 6.5)^2 + 2.25$$

Choosing another coordinate, $(0, 0)$

$$0 = a(0 - 6.5)^2 + 2.25$$

$$a = \frac{-2.25}{42.25}$$

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$$y = -0.0533(x - 6.5)^2 + 2.25$$

This gives:



C2. Use the Desmos regression tool to find a quadratic regression line. Compare this regression line to your answer for (2).

$$y_1 \sim a(x_1 - p)^2 + q$$

STATISTICS
$R^2 = 0.9988$

RESIDUALS
e_1

PARAMETERS
$a = -0.0497871$
$p = 6.64281$
$q = 2.24077$

This gives:



This clearly fits our data points very well – and better than our attempt on the right hand side.

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C3. Compare the 3 bridges – which was best described by a quadratic curve? Research why a quadratic curve was used in many bridge designs.

Both the Clifton Suspension bridge and the Mathematical bridge were well represented by a quadratic curve – even though the suspension bridge will actually follow a Catenary curve. The quadratic did not full capture the curve for the moon bridge which was clearly better represented by a half circle.

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